Short Papers

Theorems on Match and Isolation in Multiport Networks

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***Abstract*—Two theorems on match and isolation among a number of ports of a multiport network are derived. Applications are given for n-way hybrid power dividers with matched and isolated output ports.**

Theorem 1

If n ports of a lossless, but not necessarily reciprocal, $(n+m)$-port network are matched and isolated from one another, then m cannot be less than n.

The proof of this theorem is given at the end of the paper. It is understood that all ports of the network are terminated in impedances with finite resistive components, and that the said match and isolation are obtained for a given set of these impedances.

An equivalent statement of Theorem 1 follows.

Theorem 1′

If the n ports of a passive, but not necessarily reciprocal, n-port network are matched and isolated from one another, then the network contains at least n dissipative elements, such as resistors or terminations.

The equivalence of the two theorems follows by treating the dissipative elements, say m in number, of the passive network in Theorem 1′ as m ports, thus obtaining the lossless $(n+m)$-port network in Theorem 1.

A useful application of Theorem 1′ is in the area of n-way hybrid power dividers where a signal applied to an input port is divided, equally or unequally, among n output ports. It is often required that the output ports be matched and isolated from one another to eliminate interactions among devices that are connected to them. Because of the resistive component of the source impedance connected to the input port, the following is true from Theorem 1′.

Corollary 1

An n-way hybrid power divider whose output ports are matched and isolated from one another requires at least $n-1$ isolation resistors or terminations.

Indeed, for any binary $(n=2)$ hybrid power divider, such as the magic T [1], the ring hybrid [1], the branch-line hybrid [1], the Wilkinson 2-way hybrid [2], the Parad and Moynihan hybrid [3], or any directional coupler, one $(1=n-1)$ isolation resistor or termination is needed in addition to the input port. Furthermore, since an n-way power divider can be constructed by a branching cascade of $n-1$ binary hybrids, $n-1$ isolation resistors or terminations, one per binary hybrid, are required. However, because of special topology or symmetry requirements in some n-way hybrids, more than $n-1$ isolation resistors or terminations are often needed. This is the case, for example, in the n-way hybrid power dividers of Wilkinson [2] $(n \geq 3)$ and Gysel [4] $(n \geq 2)$ where n isolation resistors or terminations are used. Also, the n-way hybrid power dividers discussed by Nagai, Maekawa and Ono [5] require considerably more than $n-1$ isolation resistors to obtain the desired match and isolation.

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The following theorem was obtained as a by-product of the proof of Theorem 1.

Theorem 2

If n ports of a lossless, but not necessarily reciprocal, $2n$-port network are matched and isolated from one another, then the remaining n ports are also matched and isolated from one another.

For $n=1$, this theorem reduces to the well-known fact that if a lossless 2-port network is matched at one port, then it is also matched at the other port. A proof of Theorem 2 is available [1, pp. 300–301] for $n=2$ for the special case of reciprocal 4-port directional couplers.

A useful implication of Theorem 2 that is related to Corollary 1 follows.

Corollary 2

If an n-way power divider whose output ports are matched and isolated from one another contains $n-1$ isolation resistors or terminations, then the input port and the isolation resistors or terminations are all matched and isolated from one another.

Actually, if the power divider contains more than $n-1$ isolation resistors or terminations, then it follows from Theorem 1, with the role of n and m interchanged, that the input port of the divider and its isolation resistors or terminations cannot all be matched and isolated from one another.

Proof of Theorem 1: Consider a lossless $(n+m)$-port network that may or may not be reciprocal. Let the first n ports be treated as one group, and the last m ports as another group. Thus, the scattering matrix of the network, normalized to its port impedances, can be put in the form:

$$S=\begin{bmatrix} N & K \\ L & M \end{bmatrix} \tag{1}$$

where N, M, K, and L are $n\times n$, $m\times m$, $n\times m$, and $m\times n$ matrices, respectively. It follows from losslessness that [1, pp. 148–149]

$$S^{\dagger}S=\mathbf{1} \tag{2}$$

where the dagger † represents complex conjugation and transposition, and $\mathbf{1}$ is the unity matrix.

Suppose that the first n ports are matched and isolated from one another, i.e., that

$$N=0 \tag{3}$$

where $\mathbf{0}$ is the zero matrix. Thus it can be shown from (2) that

$$L^{\dagger}L=\mathbf{1} \tag{4}$$

$$L^{\dagger}M=\mathbf{0}. \tag{5}$$

Recall the following properties of the *rank* of matrices [6].

(1) The rank of the unit matrix is equal to its dimension.

(2) The rank of a matrix does not change by complex conjugation or by transposition.

(3) The rank of a matrix cannot exceed its smaller dimension.

(4) The rank of the product of two matrices cannot exceed the rank of either of them. Since L has the dimension of $m \times n$, it follows from (4) that

$$n=\text{rank } (L^{\dagger}L) \leqslant \text{rank } (L) \leqslant \min (m,n). \tag{6}$$

Thus

$$m \geqslant n \tag{7}$$

which proves Theorem 1.

Proof of Theorem 2: If $m=n$, then L is a square matrix. It follows from (4) that L is unitary, and hence, nonsingular. Thus (5) gives

$$M=\mathbf{0} \tag{8}$$

which proves Theorem 2.

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Schottky Barrier Impedance Measurements at UHF

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***Abstract*—The observed frequency dependence of the real part of the small signal impedance of Schottky barrier varactor diodes has previously been explained via physical phenomena. A detailed experimental investigation shows that the frequency dependence is due to inevitable systematic errors in the measurement procedure used.**

Impedance measurements made on various types of varactor diode over the past twenty years have repeatedly shown an inverse-square frequency dependence in the effective series resistance (i.e., the real part of the impedance) [1]–[5]. Some uncertainty exists over the cause of the phenomenon and this letter presents results which suggest that it is not a real effect but is due to measurement error.

The equivalent circuit for a Schottky barrier or p-n junction varactor derived from its physical structure, is a parallel combination of the junction conductance and capacitance in series with the bulk resistance. In the frequency band under consideration and in reverse bias, the junction conductance is negligible, so the effective series resistance is expected to be frequency independent up to frequencies where skin effects become significant [8]. In order to explain the observed frequency dependence previous workers have assumed that a surface inversion layer [1] or a transition region [2] exists at the contact and a parallel RC equivalent circuit has been used as a model. This form of equivalent circuit gives the observed frequency dependence since the real part of its impedance is given by

$$\text{Re}\{Z\}=\frac{1/R}{1/R^2+\omega^2C^2}.$$

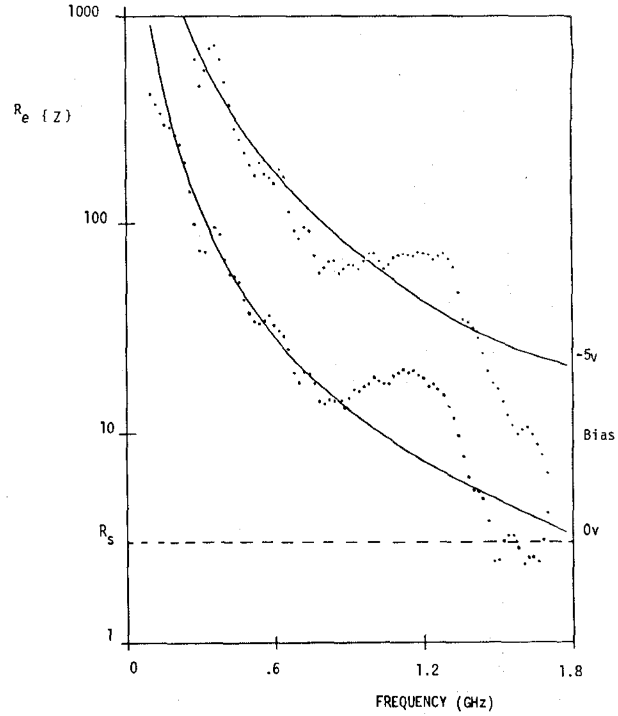

Fig. 1. Real part of the varactor impedance.···· = Measured values.—— = 0.99 reflection coefficient representing a maximum 1-percent error.

In this work, measurements have been made on Schottky barrier varactors in the frequency range 100 MHz–2 GHz. The physical structure of the devices consisted of a titanium/gold contact on a 3-μm GaAs ($N_d=10^{16}\text{cm}^{-3}$) epilayer. The device chips had a zero bias capacitance of 0.5 pF and were encapsulated in $S4$ packages. These devices were mounted in 7-mm 50-Ω silver plated coaxial line and reflection coefficient measurements made using a phase locked automatic network analyzer (HP 8542A), the reference plane being set by sliding a tight fitting copper short-circuit against the end of the inner conductor.

To extract the device-chip impedance from the measured reflection coefficient, the transfer matrix parameters of the package in this particular mount must be known. The $S4$ matrix between 2 and 18 GHz has been calculated [7] using the galvanomagnetic technique [8]. A circuit model consisting of two capacitors and an inductor in a pi configuration was fitted to the transfer matrix (since it is almost lossless and contains three independent variables), and this model was then extrapolated to UHF aided by the measured low-frequency (1-MHz) package

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